

Dark solitons in mode-locked lasers

Mark J. Ablowitz¹, Theodoros P. Horikis², Sean D. Nixon¹, Dimitri J. Frantzeskakis³

¹*Department of Applied Mathematics, University of Colorado, 526 UCB, Boulder, CO 80309-0526*

²*Department of Mathematics, University of Ioannina, Ioannina 45110, Greece*

³*Department of Physics, University of Athens, Panepistimiopolis, Zografos, Athens 15784, Greece*

Compiled December 8, 2010

Dark soliton formation in mode-locked lasers is investigated by means of a power-energy saturation model which incorporates gain and filtering saturated with energy, and loss saturated with power. It is found that general initial conditions evolve into dark solitons under appropriate requirements also met in the experimental observations. The resulting pulses are well approximated by dark solitons of the unperturbed nonlinear Schrödinger equation. Notably, the same framework also describes bright pulses in anomalous and normally dispersive lasers. © 2010 Optical Society of America

OCIS codes: 140.4050, 140.3510, 060.5530, 190.5530

Laser frequency stabilization via mode-locked (ML) lasers has become an indispensable tool in many research activities. Advances in optical frequency standards have resulted in the development of precise frequency measurement capability in the visible and near-infrared spectral regions. Although the potential for using ML lasers in optical frequency synthesis was recognized early, the available lasers did not provide the properties necessary for fulfilling this potential until recently [1]. The recent explosion of measurements based on ML lasers has been largely due to the development of the Kerr-lens mode-locked Ti:sapphire laser and its capability to generate sufficiently short pulses. Alternatively, amplified pulses of a similar duration are created through the compression of pulses via self-phase modulation in a hollow core fiber [2].

Dark solitons are intensity dips on a constant background with a phase jump across their intensity minimum. Since their discovery, cf. [3], dark solitons have attracted considerable attention, especially in the fields of nonlinear optics [4] and Bose-Einstein condensates [5]. In the non-mode-locking regime, the first train of dark solitons was successfully achieved in an all normal dispersion erbium-doped fiber laser with an in-cavity polarizer. In ML lasers they have been difficult to generate with the first experimental observation only recently reported [2]; see also [6]. Apart from the bright pulse emission, it was observed that, under strong continuous wave emission, appropriate pump strength and negative cavity feedback, a fiber laser can also emit single or multiple dark pulses [2].

To analyze dark solitons in ML lasers we use a model based on a variant of the nonlinear Schrödinger (NLS) equation, termed the power-energy saturation (PES) equation [7], suitably generalized to include non-vanishing boundary conditions at infinity. It is expressed

in the following dimensionless form,

$$i\psi_z - \frac{1}{2}\psi_{tt} + |\psi|^2\psi = \frac{ig}{1 + E/E_0}\psi + \frac{i\tau}{1 + E/E_0}\psi_{tt} - \frac{il}{1 + P/P_0}\psi, \quad (1)$$

where the complex electric field envelope $\psi(z, t)$ is subject to the boundary conditions $|\psi(z, t)| \rightarrow |\psi_\infty|$ as $|t| \rightarrow \infty$. Here, $E(z) = \int_{-\infty}^{+\infty} (|\psi_\infty|^2 - |\psi|^2) dt$ is the dark-pulse energy, $P(z, t) = |\psi_\infty|^2 - |\psi|^2$ is the instantaneous power, while E_0 and P_0 are related to the saturation energy and power, respectively. Furthermore, g , τ , and l are all positive, real constants, with the corresponding terms representing saturable gain, spectral filtering, and saturable loss. Typical dimensional numbers can be found in Ref. [8].

The above definition of the pulse energy, although unconventional, corresponds to the physical properties of the system. Indeed, as the peak-to-background intensity ratio can achieve large values in bright pulses, so can the focused-to-unfocused vacuum ratio in dark pulses. As such, dark pulses can be thought of as focusing the vacuum [9]. The definition of power follows consistently from $E = \int_{-\infty}^{+\infty} P dt$. A modified model (yielding similar results), with more standard definitions of energy and power, is also discussed below.

To study the dynamics and mode-locking capabilities of our model, we integrate Eq. (1) with a fourth-order Runge-Kutta method, with initial profile $\psi(0, t) = \tanh t$, though others, with a π -phase jump, can be used as well. The gain parameter g is varied, while $E_0 = P_0 = 1$ and $\tau = l = 0.1$. In the top panel of Fig. 1 we show the evolution of the background amplitude, which sets the dark soliton amplitude, for different values of the gain parameter g . Locking onto stable dark solitons is only achieved when the gain term is sufficiently strong, i.e. the parameter g is large enough to counter balance the losses. Thus, for $g = 0.1$ (see dashed line in the top panel of Fig. 1), the dark soliton decays quickly due to excessive loss. On the other hand, for $g = 0.2$, the

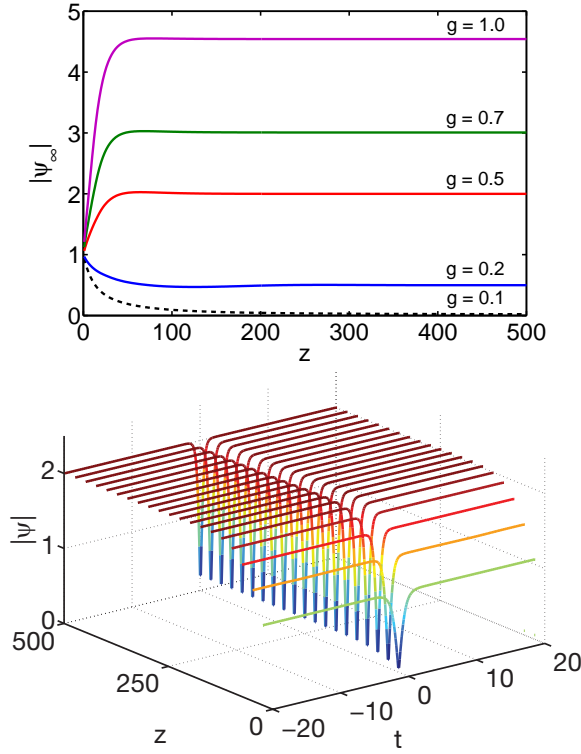


Fig. 1. (Color online) Top panel: evolution of the pulse background of an arbitrary initial profile under the PES equation with different values of gain. Bottom panel: complete evolution of the initial profile for $g = 0.5$.

pulse initially decreases, but then locks to a specific amplitude and width. Finally, for $g = 0.5, 0.7, 1.0$, a stable evolution characterized by an increase of the soliton amplitude is obtained almost instantaneously; see bottom panel of Fig. 1.

To determine the evolution of the background wave in the framework of Eq. (1), we assume that $\psi(z) = \psi_\infty(z) \exp[i\theta(z)]$. Separating real and imaginary parts, yields the following equation for the background amplitude $\psi_\infty(z)$:

$$\frac{d\psi_\infty}{dz} = \frac{g}{1 + 2|\psi_\infty|/E_0} \psi_\infty - l\psi_\infty. \quad (2)$$

Here, an approximate solution of Eq. (1) in the form $|\psi(z, t)| = |\psi_\infty \tanh(\psi_\infty t)|$ is assumed, which gives $E = 2|\psi_\infty|$. Equation (2) also describes the evolution of ψ_∞ , as depicted in the top panel of Fig. 1, and illustrates the importance of energy saturation. Indeed, if we consider the so-called similariton supporting equation (NLS with linear gain) corresponding to $\tau = l = 0$ [8, 10], with energy saturation absent ($2|\psi_\infty|/E_0 = 0$), the pulse amplitude grows exponentially at a rate defined by the gain g . Contrary, when saturation is present, this exponentially growing state cannot be reached. In fact, all terms in Eq. (1) are important for successful mode-locking. Spectral filtering is important in maintaining a short pulse duration, which is a generic phenomenon to mode-locking in

the normal dispersion regime [11]. With spectral filtering in the cavity, stable ultrashort pulses can be generated at very high values of the group-velocity dispersion. Saturable loss results in higher net gain for a train of short pulses (compared to continuous-wave operation), which is also necessary for mode-locking. In some models, loss is introduced in the form of fast saturable power absorbers placed periodically [12]. It has been shown that the lumped and distributive models yield similar results [13].

A stable equilibrium (attractor) exists for Eq. (2) and can be found setting $d\psi_\infty/dz = 0$, namely,

$$|\psi_\infty| = \frac{E_0}{2} \left(\frac{g}{l} - 1 \right). \quad (3)$$

Equation (3) is the resulting background amplitude of the dark soliton and agrees with direct numerical simulation, as seen in Fig. 2. Thus, dark solitons tend to an equilibrium (mode-lock) with constant energy and background.

The above findings agree with the experiment of Ref. [2], where the mode-locked dark pulses were identified as NLS dark solitons. In particular, they are *black* solitons (stationary kinks) characterized by a π -phase jump across the soliton notch and they can be described analytically by a hyperbolic tangent profile, i.e., $\psi(z, t) = \psi_\infty \tanh(\psi_\infty t)$. In Fig. 2 we plot the black solitons of the PES and NLS equations (for the same value of ψ_∞ , as determined from Eq. (3)) for different values of g . The amplitudes match so closely that they are in-

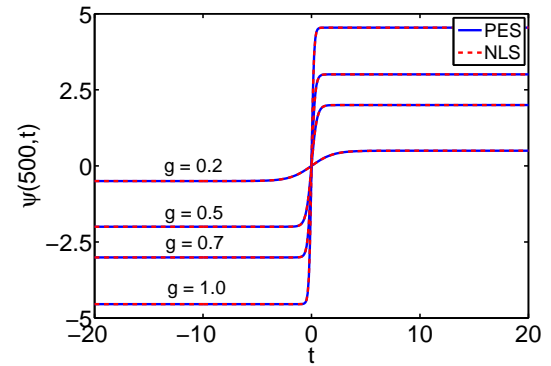


Fig. 2. (Color online) Black solitons of the PES and NLS equations for different values of g .

distinguishable in the figure, meaning that the combined perturbation of gain, filtering and loss in the PES model underlies the mode-locking mechanism: its effect is to mode-lock to a black soliton of the pure NLS equation with the appropriate background, Eq. (3). On the other hand, mode-locking to *grey* solitons, namely moving dark pulses with a phase-jump less than π at their (non-zero) intensity minima, are not found in this model as they have insufficient dark-pulse energy. This too agrees with Ref. [2] where grey solitons were not observed. Note that recent experimental [14] and theoretical [15] results in

ML lasers in the anomalous dispersion regime indicate that the normalized intensity of a pulse in a ML laser can be described by the bright solitons of the unperturbed NLS equation.

The conventional definition of the pulse energy may also be used in Eq. (1), namely $E = \frac{1}{T} \int_{-T/2}^{T/2} |\psi|^2 dt$, where T is the averaged time through the cavity. In this case, looking for stationary solutions of Eq. (3) yields

$$|\psi_\infty|^2 = \frac{g - l}{l/E_0 - g/P_0} \quad (4)$$

for the background where, for sufficiently large T , $E = \frac{1}{T} \int_{-T/2}^{T/2} |\psi|^2 dt \approx \frac{1}{T} |\psi_\infty|^2 T = |\psi_\infty|^2$. Equation (4) is a more restrictive condition for mode-locking which can also be attributed to the experimental difficulties of the problem. Furthermore, this definition results in more complicated dynamics and the development of a *shelf* on the soliton background, which is more pronounced as compared to the previous case. In fact, as shown elsewhere [16], shelves occur naturally in the perturbation theory for dark solitons. We illustrate the resulting shelf in Fig. 3, where now $g = 0.2$, $l = 0.1$, $E_0 = 1$ and $P_0 = 5$. Due to the noisy background this feature maybe difficult

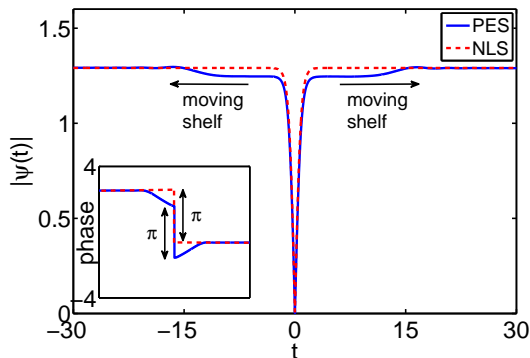


Fig. 3. (Color online) The development of a shelf in the PES equation (solid line) as compared to the solution of the NLS equation (dashed line).

to observe in an experiment. However, the shelf also affects the phase of the resulting pulse (see inset in Fig. 3) and since the experimentally observed pulses exhibit a sharp π -phase jump [2], the first dark-pulse energy definition may give a closer approximation for this problem.

Lastly, we mention the notion of multiple dark solitons. To observe these, we evolve Eq. (1) with initial condition $\psi(0, t) = \tilde{\psi}_\infty \tanh(\tilde{\psi}_\infty t + t_0)$ for $t < 0$ and $\psi(0, t) = -\tilde{\psi}_\infty \tanh(\tilde{\psi}_\infty t - t_0)$ for $t > 0$, where, $\tilde{\psi}_\infty$ can be approximated by $\psi_\infty/2$; the dark-pulse energy is now twice that of the single soliton. In Fig. 4 we show its evolution. As in the case of bright pulses [17], the two solitons are taken initially sufficiently far apart (about 10 times their full-width of half-maximum) to minimize interactions. The major difference with the bright case, where the interaction is logarithmically slow [17], is that

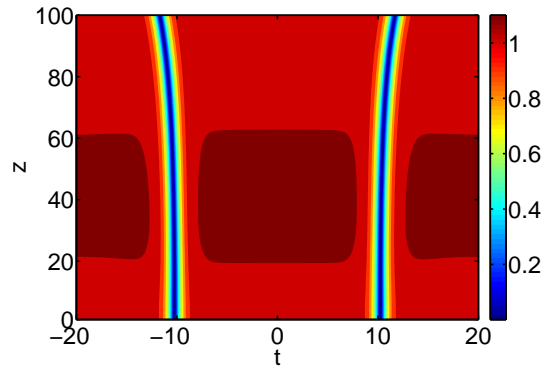


Fig. 4. (Color online) Two-dark-soliton evolution.

pulses repel sooner here due to the shelf interactions. While the solitons are stable for reasonably long distance enough to also be experimentally observed, nevertheless one can expect this case to be sensitive as with the single dark soliton.

In conclusion, we presented a power-energy saturation model that describes mode-locking of dark solitons in lasers. Much like their bright counterparts, these pulses (as well as multiple pulses) exist when sufficient gain is present in the system, which agrees with the experimental observations. The specific energy saturated gain and filtering, and power saturated loss, are crucial to the mode-locking mechanism. The resulting pulses are essentially modes of the unperturbed NLS equation with background amplitudes appropriately defined by the gain and loss parameters.

This research was partially supported by the U.S. Air Force Office of Scientific Research, under grant FA9550-09-1-0250.

References

1. S. T. Cundiff, J. Ye, and J. L. Hall, “Optical frequency synthesis based on mode-locked lasers,” *Rev. Sci. Instrum.* **72**, 3749–3771 (2001).
2. H. Zhang, D. Y. Tang, L. M. Zhao, and X. Wu, “Dark pulse emission of a fiber laser,” *Phys. Rev. A* **80**, 045803 (2009).
3. A. Hasegawa and F. Tappert, “Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. II. Normal dispersion,” *Appl. Phys. Lett.* **23**, 171–172 (1973).
4. Y. S. Kivshar and B. Luther-Davies, “Dark optical solitons: physics and applications,” *Phys. Rep.* **298**, 81–197 (1998).
5. D. J. Frantzeskakis, “Dark solitons in atomic Bose-Einstein condensates: from theory to experiments,” *J. Phys. A: Math. Theor.* **43**, 213001 (2010).
6. M. Feng, K. L. Silverman, R. P. Mirin, and S. T. Cundiff, “Dark pulse laser,” preprint (2010).
7. M. J. Ablowitz, T. P. Horikis, and B. Ilan, “Solitons in dispersion-managed mode-locked lasers,” *Phys. Rev. A* **77**, 033814 (2008).
8. M. J. Ablowitz and T. P. Horikis, “Solitons in normally

- dispersive mode-locked lasers,” *Phys. Rev. A* **79**, 063845 (2009).
9. G. Vople, “The dark side of lasers,” *Optics & Photonics Focus* **8**, story 5 (2010).
 10. S. Boscolo, S. K. Turitsyn, V. Y. Nonokshenov, and J. H. B. Nijhof, “Self-similar parabolic optical solitary waves,” *Th. Math. Phys.* **133**, 1647–1656 (2002).
 11. B. G. Bale, J. N. Kutz, A. Chong, W. H. Renninger, and F. W. Wise, “Spectral filtering for mode locking in the normal dispersive regime,” *Opt. Lett.* **33**, 941–943 (2008).
 12. F. O. Ilday, J. R. Buckley, W. G. Clark, and F. W. Wise, “Self-similar evolution of parabolic pulses in a laser,” *Phys. Rev. Lett.* **92**, 213901 (2004).
 13. M. J. Ablowitz and T. P. Horikis, “Solitons and spectral renormalization methods in nonlinear optics,” *Eur. Phys. J. Special Topics* **173**, 147–166 (2009).
 14. S. T. Cundiff, “Soliton dynamics in mode-locked lasers,” *Lect. Notes Phys.* **661**, 183–206 (2005).
 15. M. J. Ablowitz and T. P. Horikis, “Pulse dynamics and solitons in mode-locked lasers,” *Phys. Rev. A* **78**, 011802 (2008).
 16. M. J. Ablowitz, T. P. Horikis, S. D. Nixon, and D. J. Frantzeskakis, “Perturbations of dark solitons,” *arXiv:1008.3756v1* (2010).
 17. M. J. Ablowitz, T. P. Horikis, and S. D. Nixon, “Soliton strings and interactions in mode-locked lasers,” *Opt. Comm.* **282**, 4127–4135 (2009).